2.1 Product and Quotient Rules: Form A Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_form.py`). All problems are original; provenance is in `PROVENANCE_FORMS.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Simplifying first, the product rule, and the quotient rule are all accepted wherever they apply; no method is required, and long but correct work gets full credit.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part unless a row says otherwise.
- **Domains.** On Q1, a correct note on where f' does not exist is accepted, never required.
- **Simplifying.** Answers need not be simplified beyond what a row says; equivalent forms are correct.

Form A

Item	Answer	Rubric / trap
1(a)	$4x^3 \sin x + x^4 \cos x$	product rule: $(4x^3)(\sin x) + (x^4)(\cos x)$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* multiplying the derivatives is not a rule; earns 0.
1(b)	$6x^2 + 10x - 12$	product rule $2(x^2+4x)+(2x-3)(2x+4)$, or expand to $2x^3+5x^2-12x$ first. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* $(2)(2x+4)$, the product of the derivatives, earns 0.
1(c)	$\dfrac{4x}{(x^2+1)^2}$	quotient rule: $\dfrac{2x(x^2+1)-(x^2-1)(2x)}{(x^2+1)^2}$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* the numerator is not 0: $2x^3+2x-2x^3+2x = 4x$; reversing it gives $-4x$.
1(d)	$\frac{5}{2}x^{3/2} - 2x^{-1/2}$	simplify first: $x^{5/2} - 4x^{1/2}$. The quotient rule also works. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* dividing only the first term by $\sqrt{x}$; $4 \cdot \frac{1}{2} = 2$.
1(e)	0 (for every x in the domain of f)	$\dfrac{4x(3x+2)}{x(3x+2)} = 4$ wherever f is defined, a constant. The quotient rule, carried through, also gives 0. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* a constant in disguise; the quotient rule is valid but long.
1(f)	$-5\csc^2 x$	$\sin x \csc x = 1$ wherever $\csc x$ is defined, so $f(x) = 1 + 5\cot x$. The product rule on $\sin x \csc x$ also gives 0. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* a constant hidden in a product; the derivative of $\cot x$ is $-\csc^2 x$, with a minus sign.
1(g)	$\sec x + x \sec x \tan x$	product rule: $(1)(\sec x) + (x)(\sec x \tan x)$. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* $\sec x \tan x$ alone drops the first term.
1(h)	0	$\cot\left(\frac{\pi}{4}\right) = 1$ and $\csc\left(\frac{\pi}{6}\right) = 2$ are numbers, so $f(x) = \frac{1}{2}$ for every x. 1 pt rule applied or simplification, 2 pts correct derivative. *Trap:* no x appears; the quotient rule applied as if these were functions of x earns 0.

Item	Answer	Rubric / trap
2	-2	2 pts $f'(x) = \dfrac{\cos x(1-\cos x) - \sin x \cdot \sin x}{(1-\cos x)^2} = \dfrac{\cos x - 1}{(1-\cos x)^2} = \dfrac{-1}{1-\cos x}$; 2 pts substitution $\cos\frac{\pi}{3} = \frac{1}{2}$; 1 pt value. *Trap:* $\sin^2 x + \cos^2 x = 1$ collapses the numerator; reversing it gives $+2$; the derivative of $1 - \cos x$ is $+\sin x$.
3(a)	2	the h-form of $f'\left(\frac{\pi}{4}\right)$ for $f(x) = \tan x$, since $\tan\frac{\pi}{4} = 1$; $\sec^2\frac{\pi}{4} = 2$. 1 pt naming f and c (or any valid evaluation), 2 pts value. *Trap:* the 1 is $\tan\frac{\pi}{4}$ in disguise; plugging in $h = 0$ gives $\frac{0}{0}$, not an answer.
3(b)	0	the c-form of $f'(\pi)$ for $f(x) = \sec x$, since $\sec\pi = -1$; $\sec\pi\tan\pi = (-1)(0) = 0$. 1 pt naming f and c (or any valid evaluation), 2 pts value. *Trap:* $+1$ is $-\sec\pi$ in disguise; the answer 0 is correct, not a sign of an error.
4	$y - \frac{1}{2} = \frac{3}{4}(x-1)$, i.e. $y = \frac{3}{4}x - \frac{1}{4}$	1 pt $f(1) = \frac{1}{2}$; 2 pts $f'(x) = \dfrac{2x(x+1) - x^2}{(x+1)^2}$; 1 pt $f'(1) = \frac{3}{4}$; 2 pts equation, any correct form. *Trap:* using $f'(1)$ as the y-coordinate; reversed numerator gives slope $-\frac{3}{4}$.
5	$-\dfrac{\pi^2}{8}$	1 pt $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$; 2 pts $f'(x) = \tan x + x\sec^2 x$, so $f'\left(\frac{\pi}{4}\right) = 1 + \frac{\pi}{2}$; 2 pts set $x = 0$ in $y - \frac{\pi}{4} = \left(1 + \frac{\pi}{2}\right)\left(x - \frac{\pi}{4}\right)$. *Trap:* the item asks for a number, the y-intercept, not an equation; an equation alone earns 3.
6	$x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$	2 pts $f'(x) = \cos^2 x - \sin^2 x$; 2 pts $\tan^2 x = 1$, $\tan x = \pm 1$; 2 pts both solutions in the interval and nothing else. *Trap:* $\tan x = 1$ alone gives only $\frac{\pi}{4}$; $\tan x = -1$ gives $\frac{3\pi}{4}$, also in $(0, \pi)$.
7	$\sec x \tan x$ (shown)	1 pt rewrite $\sec x = \dfrac{1}{\cos x}$; 2 pts quotient rule $\dfrac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x}$; 2 pts $= \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}$; 1 pt $= \sec x \tan x$. *Trap:* quoting the sec rule earns 0; the derivative of the constant 1 is 0; $-(-\sin x) = +\sin x$.
8(a)	8	$h'(2) = f'(2)g(2) + f(2)g'(2)$, with the row $x = 2$. 1 pt rule, 2 pts entries, 1 pt value. *Trap:* $f'(2)g'(2) = -4$; $f(2)g(2) = -3$. Columns run x, $g(x)$, $f'(x)$, $g'(x)$, $f(x)$.
8(b)	$-\frac{1}{8}$	$k'(4) = \dfrac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2}$, with the row $x = 4$. 1 pt rule, 2 pts entries, 1 pt value. *Trap:* reversed numerator gives $\frac{1}{8}$; $[g(4)]^2 = 16$, not -16.
8(c)	-9	$m'(x) = f(2)\,g'(x)$ because $f(2) = -3$ is a number, so $m'(4) = f(2)\,g'(4)$, with the row $x = 4$. 2 pts recognizing the constant multiple, 2 pts value. *Trap:* $f(2)$ is a number; the product rule with $f'(2) = 2$ gives -17; earns at most 1.
9(a)	3	$h'(x) = f'(x)g(x) + f(x)g'(x)$, $-6 = (-3)(4) + 2g'(1)$; substitute the given values and solve for $g'(1)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* the item gives the derivative of the product and asks for a factor's derivative; $h' = f'g'$ gives 2.
9(b)	1	$q'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$, $-1 = \dfrac{2f'(3) - 6}{4}$; substitute the given values and solve for $f'(3)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* reversing the numerator gives 5; $q' = f'/g'$ gives -1.
9(c)	-2	$w'(x) = f(x) + x\,f'(x)$, $4 = f(2) + 2(3)$; substitute the given values and solve for $f(2)$. 2 pts rule written with letters, 1 pt substitution, 1 pt value. *Trap:* this part asks for a function value, $f(2)$; $w' = x\,f'(x)$ (dropping $f(x)$) gives $6 = 4$, a contradiction, not an answer.
10(a)	-3	Readings: $f(-1) = -2$, $f'(-1) = 1$, $g(-1) = 1$, $g'(-1) = 2$ (slopes are rise over run of the segment containing $x = -1$). $p'(-1) = f'(-1)g(-1) + f(-1)g'(-1)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $f'(-1)g'(-1) = 2$; $f(-1)g(-1) = -2$; mixing a value from one graph with a slope from the other.

Item	Answer	Rubric / trap
10(b)	-5	Readings: $f(1) = -2$, $f'(1) = -1$, $g(1) = 1$, $g'(1) = -2$ (slopes are rise over run of the segment containing $x = 1$). $q'(1) = \dfrac{f'(1)g(1) - f(1)g'(1)}{[g(1)]^2}$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* reversed numerator gives 5; $f'(1)/g'(1) = \frac{1}{2}$; swapping the graphs gives $\frac{5}{4}$.
10(c)	3	Readings: $f(-3) = -2$, $f'(-3) = -1$, $g(-3) = 1$, $g'(-3) = -2$ (slopes are rise over run of the segment containing $x = -3$). $r'(-3) = 2(-3)f(-3) + (-3)^2 f'(-3)$. 2 pts readings, 1 pt rule, 1 pt value. *Trap:* $(-3)^2 = 9$; writing -9 gives 21; dropping the $2x\,f(x)$ term gives -9.
11	II	6 pts, all or nothing. In II the dashed curve is $f(x) = \frac{x^3}{8} - \frac{3x}{2}$ and the solid curve is $f'(x) = \frac{3x^2}{8} - \frac{3}{2}$: it is zero exactly where the dashed curve turns ($x = -2, 2$), positive where that curve rises, and negative where it falls. Wrong choices: I: the parabola is zero at $x = \pm 1$, not at the turning points $x = \pm 2$; III: shifted: the other curve is zero at $x = -1$ and $x = 3$, not at the turning points $x = \pm 2$; IV: the line is the derivative of the derivative: it is zero at $x = 0$, not where the cubic turns. *Trap:* assuming the dashed curve is always the derivative; the styles swap between graphs.

Points: $24 + 5 + 6 + 6 + 5 + 6 + 6 + 12 + 12 + 12 + 6 = 100$.